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THE DETERMINATION OF EFFECTIVE COLUMN LENGTH FROM STRAIN MEASUREMENTS

By Evan H. Schuette and J. Albert Roy

Langley Memorial Aeronautical Laboratory
Langley Field, Va.

NACA

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

ADVANCE RESTRICTED REPORT

THE DETERMINATION OF EFFECTIVE COLUMN LENGTH FROM STRAIN MEASUREMENTS

By Evan H. Schuette and J. Albert Roy

SUMMARY

A method is presented for the experimental determination of the effective length of a column for which the end conditions are unknown by establishing the points of zero curvature from readings of strain gages distributed along the length of the column. Tests of four columns of different cross sections indicated that the proposed method gives satisfactory results even when there is considerable scatter in the strain-gage readings.

INTRODUCTION

A suitable method is needed for experimentally evaluating the effective length from column tests in which the end conditions are unknown. One such method consists in establishing the deflection curve from measurements of lateral deflections and estimating from this curve the locations of the inflection points. The principal objection to this method is that the determination of inflection points from an experimentally established curve is an inherently inaccurate procedure. A more accurate method of establishing the inflection points consists in measuring the curvature rather than the deflection and establishing from such measurements the points of zero curvature, which define the inflection points. The present report shows how strain measurements can be used to indicate curvature and thus to establish the effective column length. Four columns of different cross sections were tested to provide an experimental check of the method.

SYMBOLS

L	actual length of column, inches
L_e	effective length of column, inches
ϵ_1, ϵ_2	strains at two points in cross section
d	distance between points in cross section for which strains ϵ_1 and ϵ_2 are taken, measured perpendicular to neutral axis, inches
r	radius of curvature, inches

METHOD OF MEASURING CURVATURE AND DETERMINING EFFECTIVE COLUMN LENGTH

If it is assumed that sections remain plane after bending, the curvature $1/r$ of a column at a given cross section is related to the difference in strain at two points on the particular cross section according to the equation

$$\frac{1}{r} = \frac{\epsilon_1 - \epsilon_2}{d}$$

This equation indicates a convenient experimental method for determining curvature from strain measurements taken along the length of the column. The method consists in attaching a number of strain gages on opposite sides of the column along its length and recording the differences in strain on the two sides. It is then necessary only to plot the curvature - or, if d is constant, the strain difference $\epsilon_1 - \epsilon_2$ - against distance along the length of the column and to determine the points of zero curvature.

SPECIMENS AND TEST EQUIPMENT

In order to check experimentally the practicability of the strain-measurement method for determining effective

length, columns of four different types of cross section were tested. A rectangular bar was tested with strain gages placed opposite each other on the wider sides. The cross sections and the locations of the resistance-type wire strain gages for the Z-section column, the skin-and-stiffener column, and the hat-stiffened panel column are shown in figures 1, 2, and 3, respectively.

The wire strain gages were attached along the length of each column at small intervals. The approximate distance d between strain gages at a particular cross section, measured perpendicular to the neutral axis, was:

Column	d (in.)
Bar	0.8
Z-section	.8
Skin-and-stiffener	2.0
Hat-stiffened panel	.9

The over-all accuracy of the strain measurements was within 2 percent.

The specimens were tested flat-ended in hydraulic testing machines having accuracies within three-quarters of 1 percent for the range of load used. Figure 4 shows the Z-section column under load.

RESULTS AND DISCUSSION

The results of the tests are presented in two sets of figures. The first set (figs. 5 to 8) shows the curvature of the columns, as given by the differences in strain for the gages at each cross section, plotted against distance along the length of the column for several loads near the maximum load. The horizontal lines represent zero strain difference. The second set (figs. 9 to 12) shows the ratio of effective length to actual length, obtained from the first set of figures, plotted against load.

Except for the Z-section column, the values plotted in figures 5 to 8 represent the increase of curvature

caused by loads in excess of a particular initial load near the maximum. The use of a fairly high initial load in the calculation of strain differences had the effect of eliminating a large part of the scatter in the test data. It was believed that this procedure would not affect the accuracy of the results, because the strain differences at the particular initial load were quite small in comparison with the differences that were recorded at loads near the maximum. For the Z-section (fig. 6), however, a somewhat more satisfactory plot was obtained when the strain increments were taken from zero load, because large strain differences were recorded as soon as loading was started.

A part of the scatter in the plot for the skin-and-stiffener column (fig. 7) results from the fact that slightly different curvatures were measured on the right and left sides of the column. Near the bottom of the column, especially on the left side, some of the scatter is thought to be due also to local buckling of the skin. All strain gages were placed on the stiffener, but it is probable that the effect of the buckles in the skin carried over to some extent into the stiffener and influenced the strain readings.

In the plot for the hat-stiffened panel (fig. 8) the effects of any possible differences in curvature between the two stiffeners to which strain gages were attached were eliminated by using averages of the corresponding gage readings on the two stiffeners.

Figures 5 to 8 establish the effective lengths for several loads below the maximum. In order to extrapolate from these values to the maximum load, the ratio of effective length to actual length is plotted against load in figures 9 to 12. In the extrapolation, greater weight was given to the values obtained at the higher loads. This procedure seemed justified because the absolute scatter in the curvature plots was approximately the same for all loads and thus relatively less important in comparison with the greater curvatures that existed at higher loads. Theoretical considerations, moreover, lead to the conclusion that the shape of the elastic curve for maximum load is more and more accurately approximated as the maximum load is approached, and the effective length thus approaches a definite value.

It will be noted that, for all except the Z-section column, a reasonable extrapolation to maximum load was accomplished by drawing a horizontal line through the test points. This fact is of particular interest in the case of the skin-and-stiffener column (figs. 7 and 11), for which quite consistent results were obtained even though considerable scatter was evident in the curvature plot. For the Z-section column (fig. 10), the extrapolation was accomplished by drawing a curve through the points for high loads in such a manner that the tangent to the curve was horizontal at the maximum load.

Because of the presence of ineffective widths of skin, it was impossible to check the loads obtained from the skin-and-stiffener column and the hat-stiffened panel against the values given by the Euler column formula. Such a check was made for the other two columns, however, and the results are given in the following table:

Column	Experimental maximum load (lb)	Euler load based on experimental L_e (lb)
Bar	50,000	47,800
Z-section	2,800	2,930

The calculated values of Euler load, based on the experimentally determined effective length, are within 5 percent of the test values.

LIMITATIONS OF THE METHOD

The method presented for the determination of effective column length from strain measurements is not applicable in all cases. If there is local buckling of the column, the strain measurements will be adversely affected. Whether the results are completely invalidated will depend on the extent to which the local buckling takes place at the points where the strain gages are located. In the tests of the skin-and-stiffener column reported herein the local buckling did not carry over into the sides of the stiffener, where the strain gages were located, in a sufficient degree to invalidate

completely the results obtained but the effects of the buckling were evident in the strain measurements.

Another limitation is imposed by the size of the column being tested. It is evident that the accuracy of the results depends to some extent on the distance d between strain gages. If the column cross section is such that this distance must be small, the accuracy of the results will be impaired, especially if there is a variation in strain across the width of the strain gages.

CONCLUDING REMARKS

The test data presented indicate that the proposed strain-measurement method for the experimental determination of the effective length of a column for which the end conditions are unknown gives satisfactory results even when there is considerable scatter in the strain-gage readings.

Langley Memorial Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va.

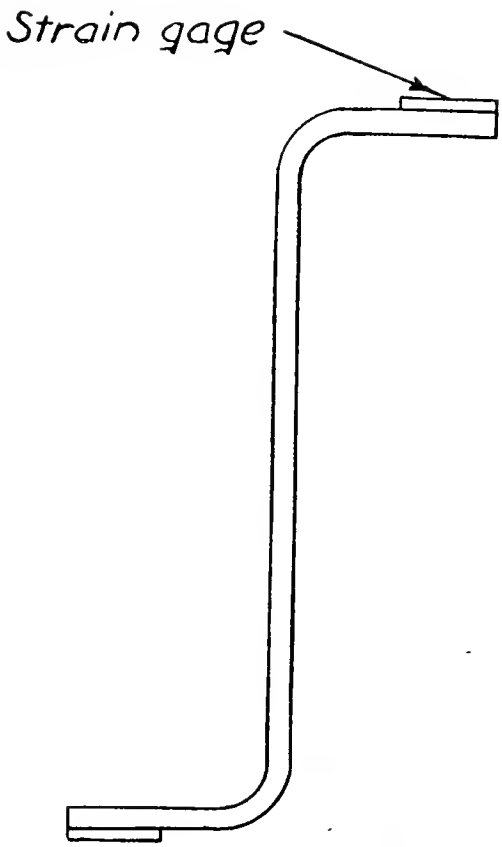

Figure 1.- Cross section of Z-section column.

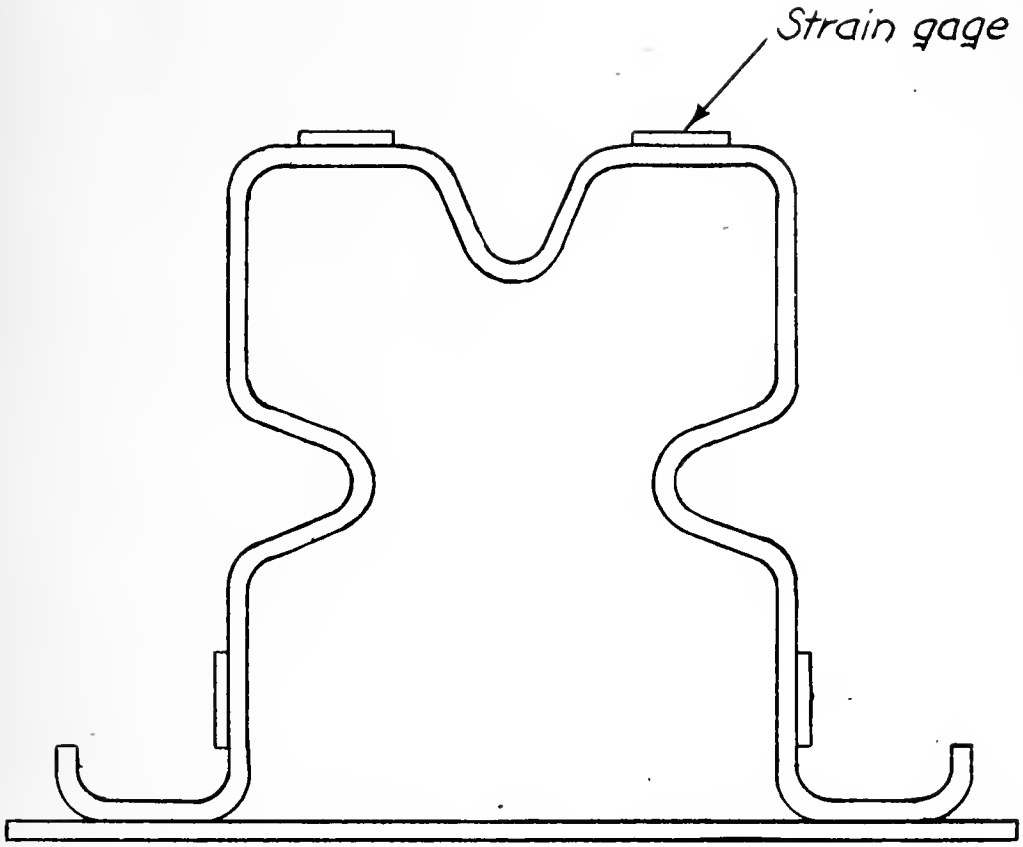

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Figure 2.- Cross section of skin-and-stiffener column.

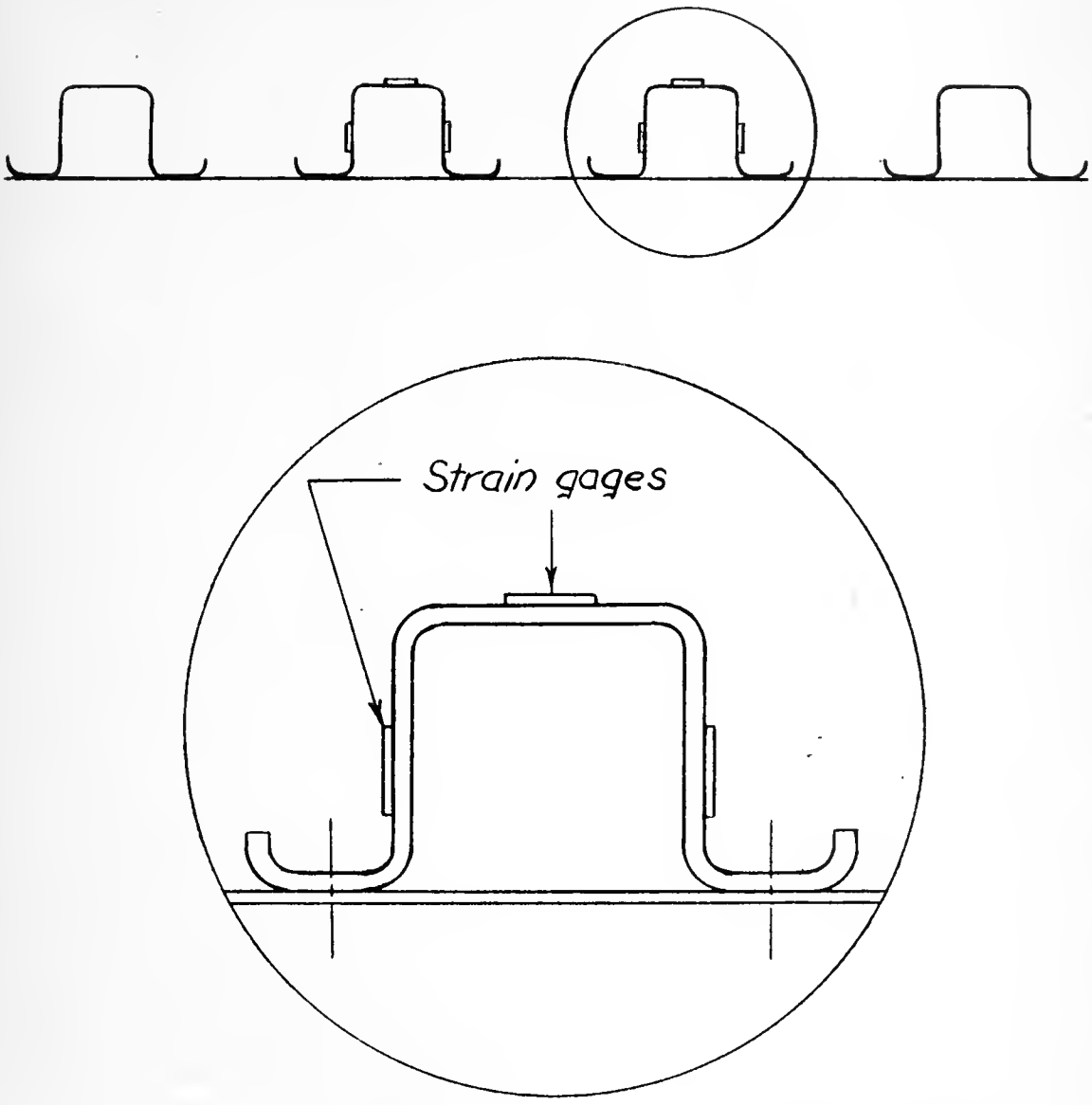


Figure 3.- Cross section of hat-stiffened panel column.

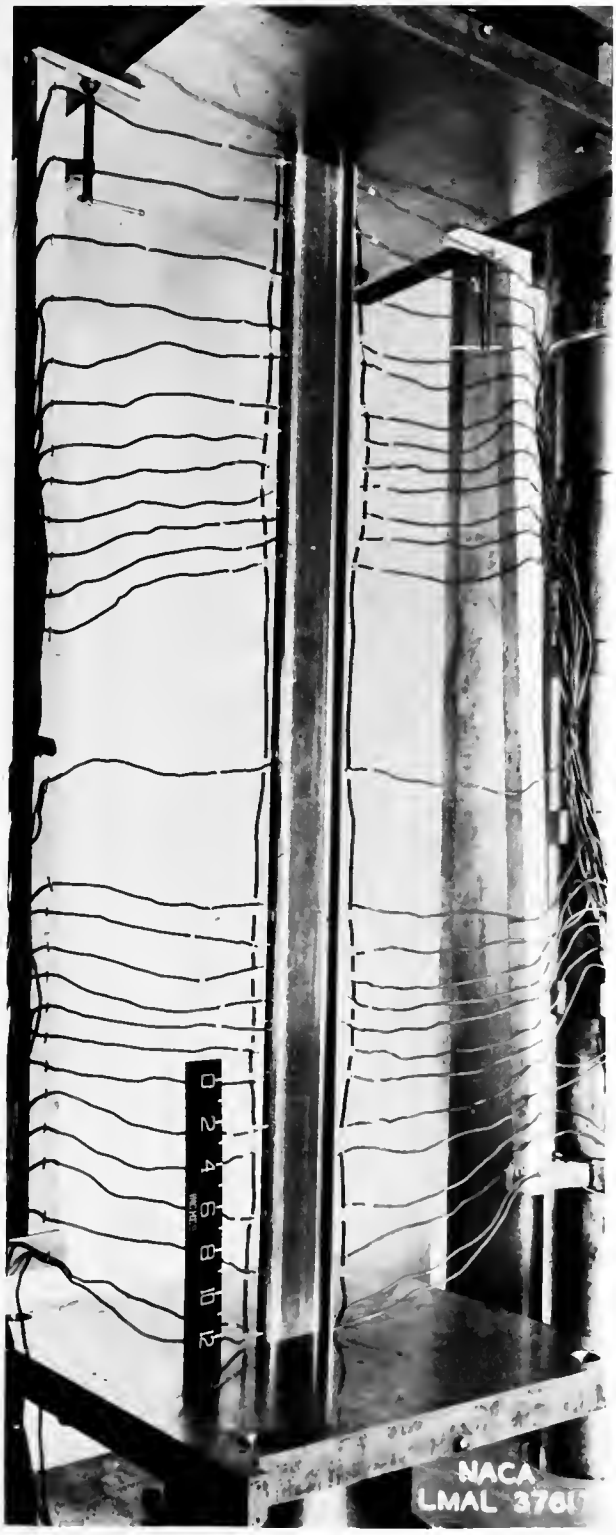


Figure 4.- Z-section column under load.

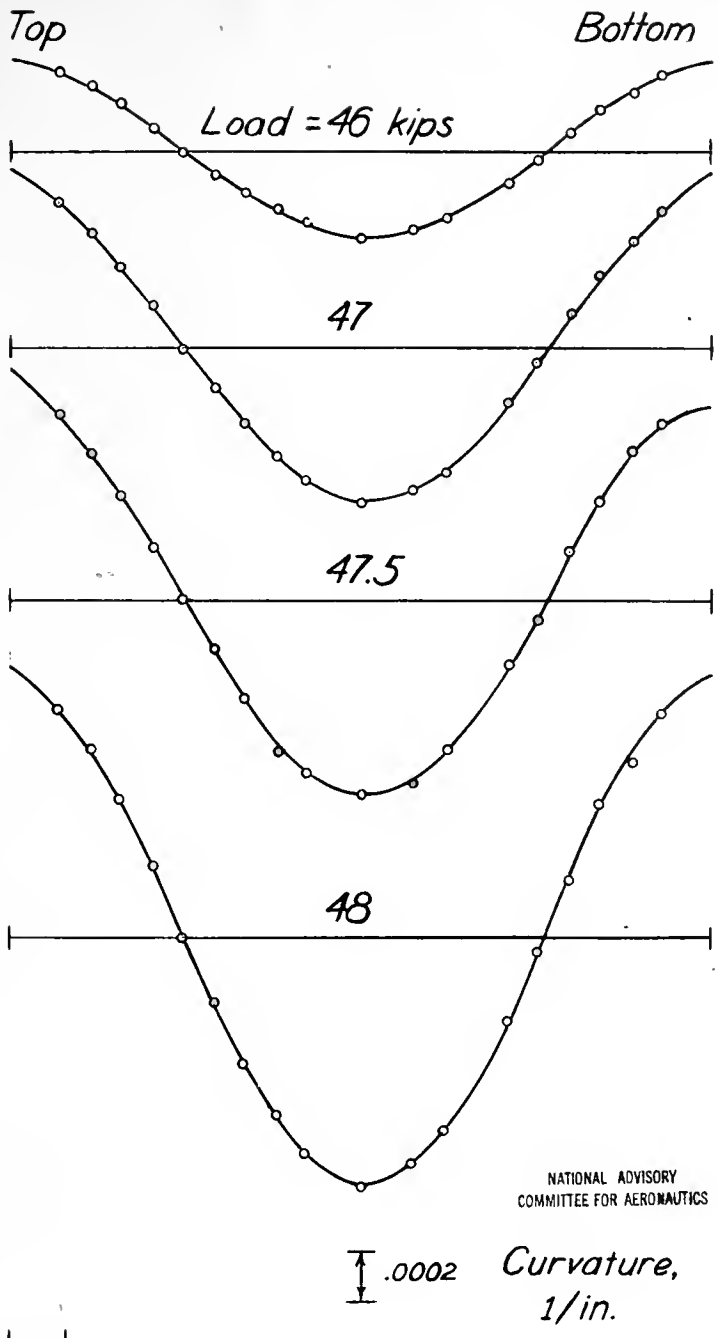


Figure 5.- Curvature of rectangular bar.

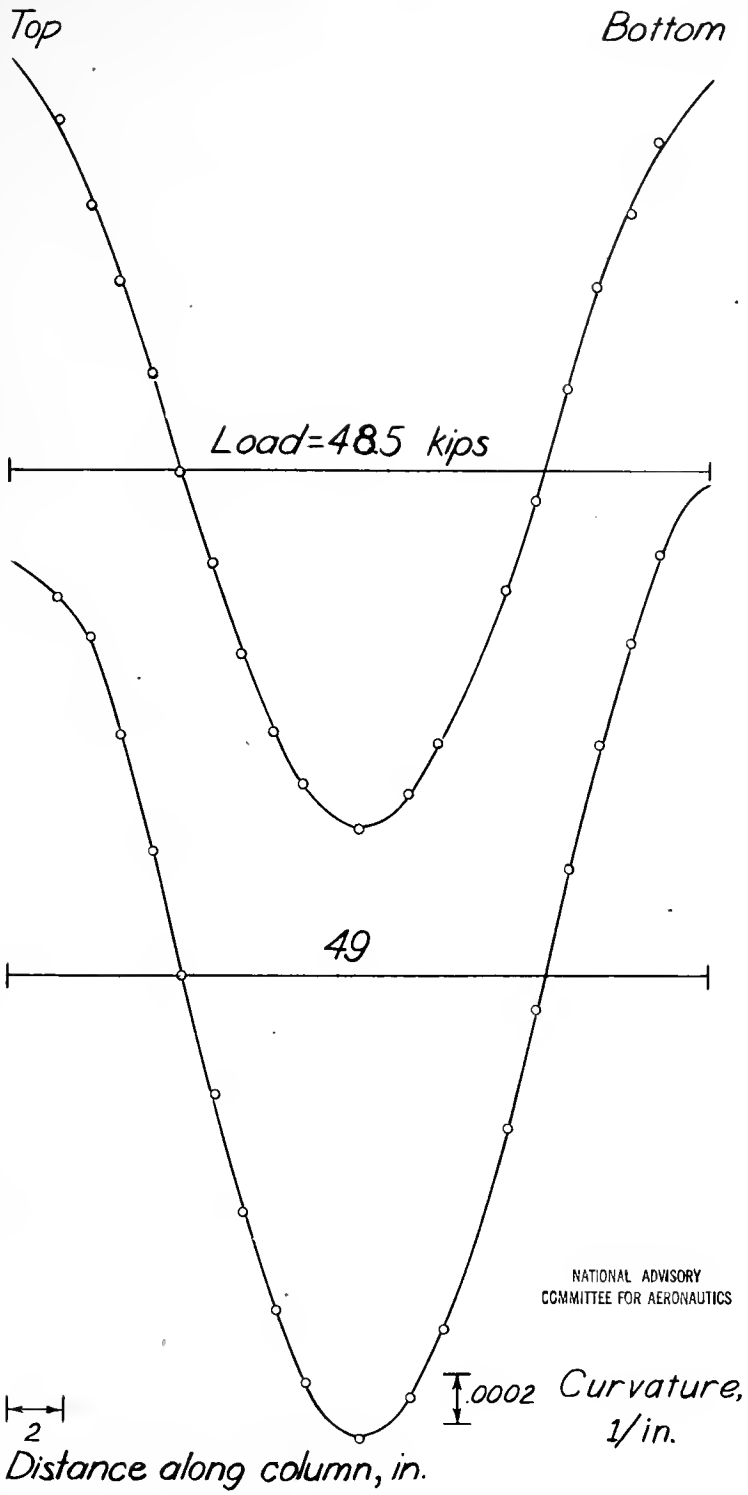
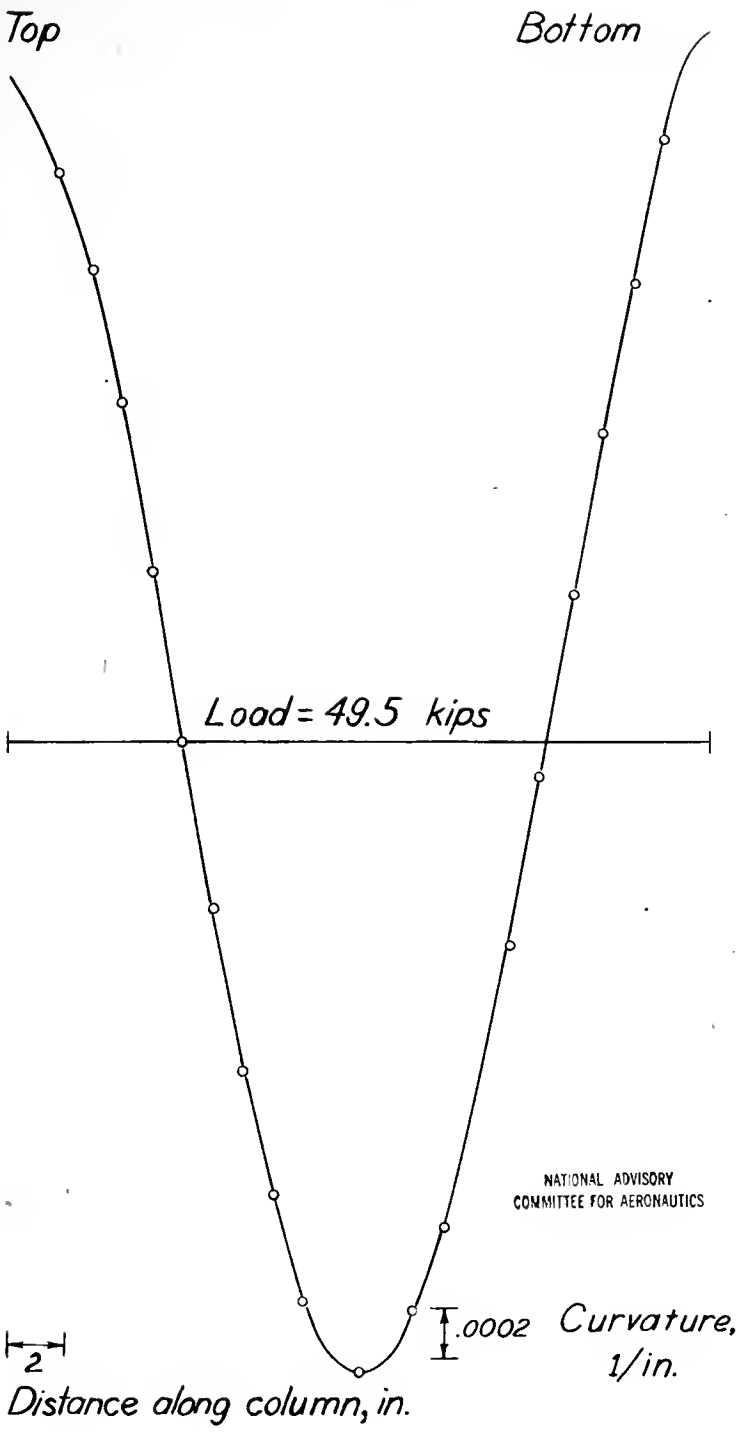


Figure 5.- Continued.



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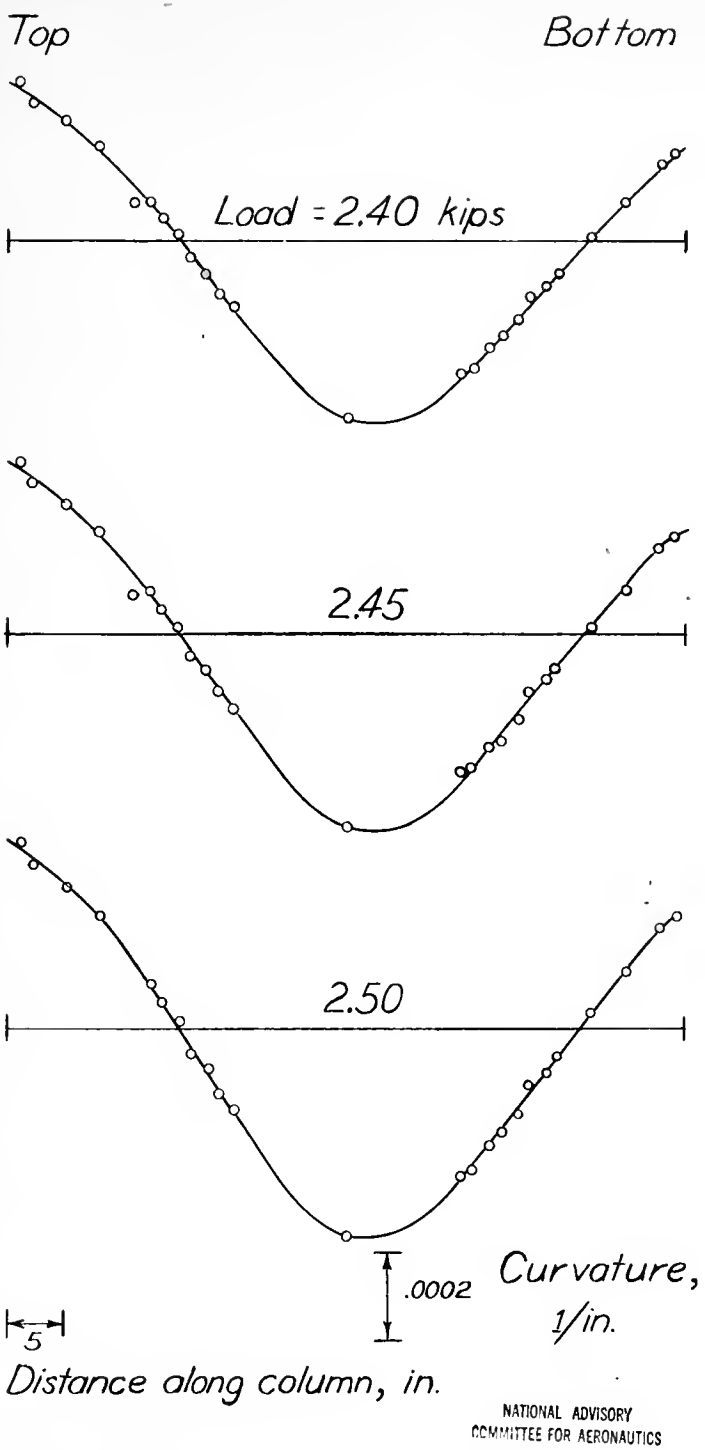
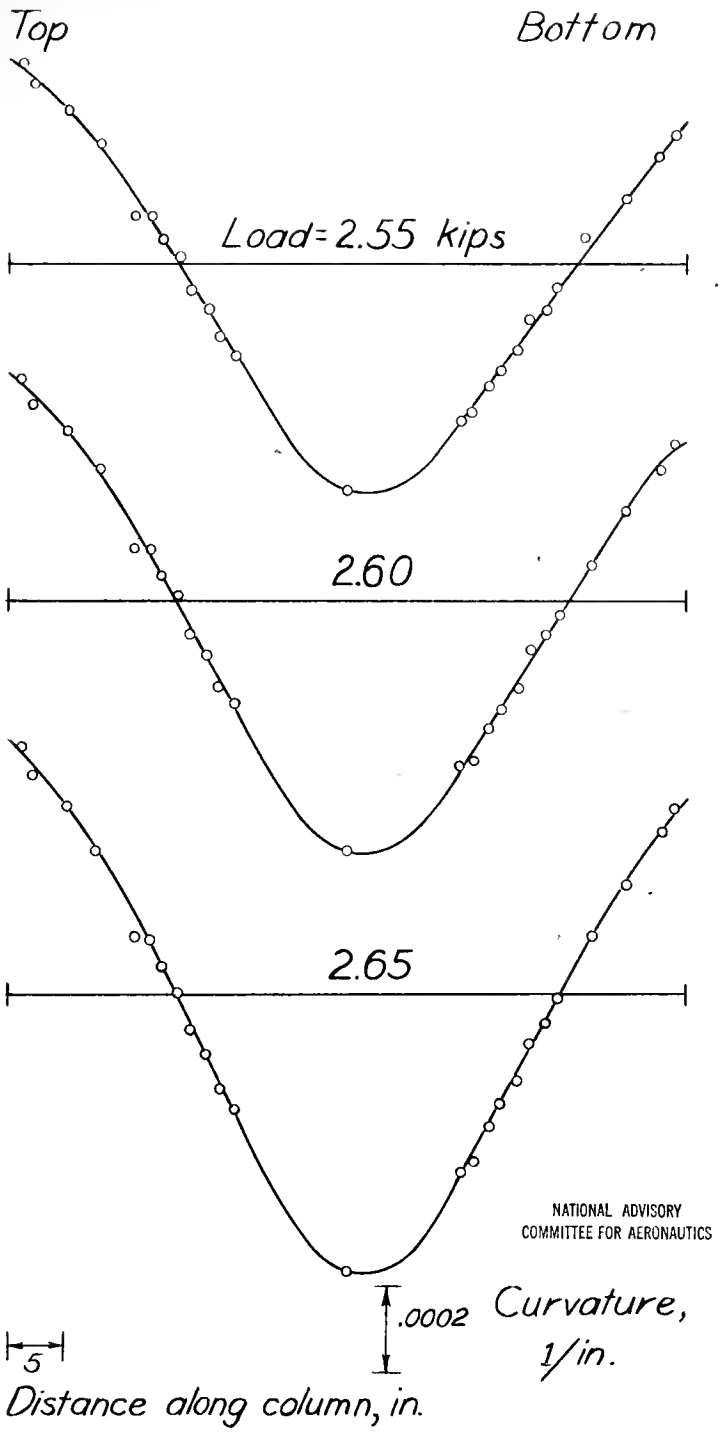


Figure 6.- Curvature of Z-section column.



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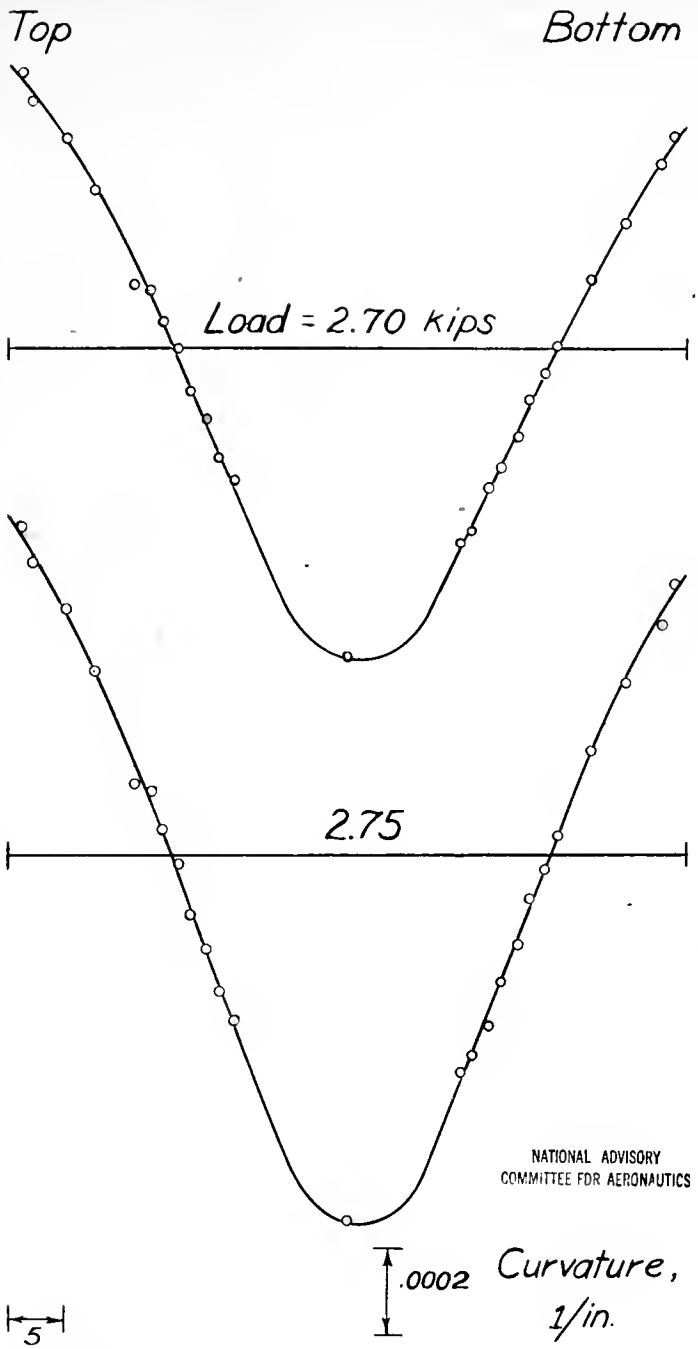

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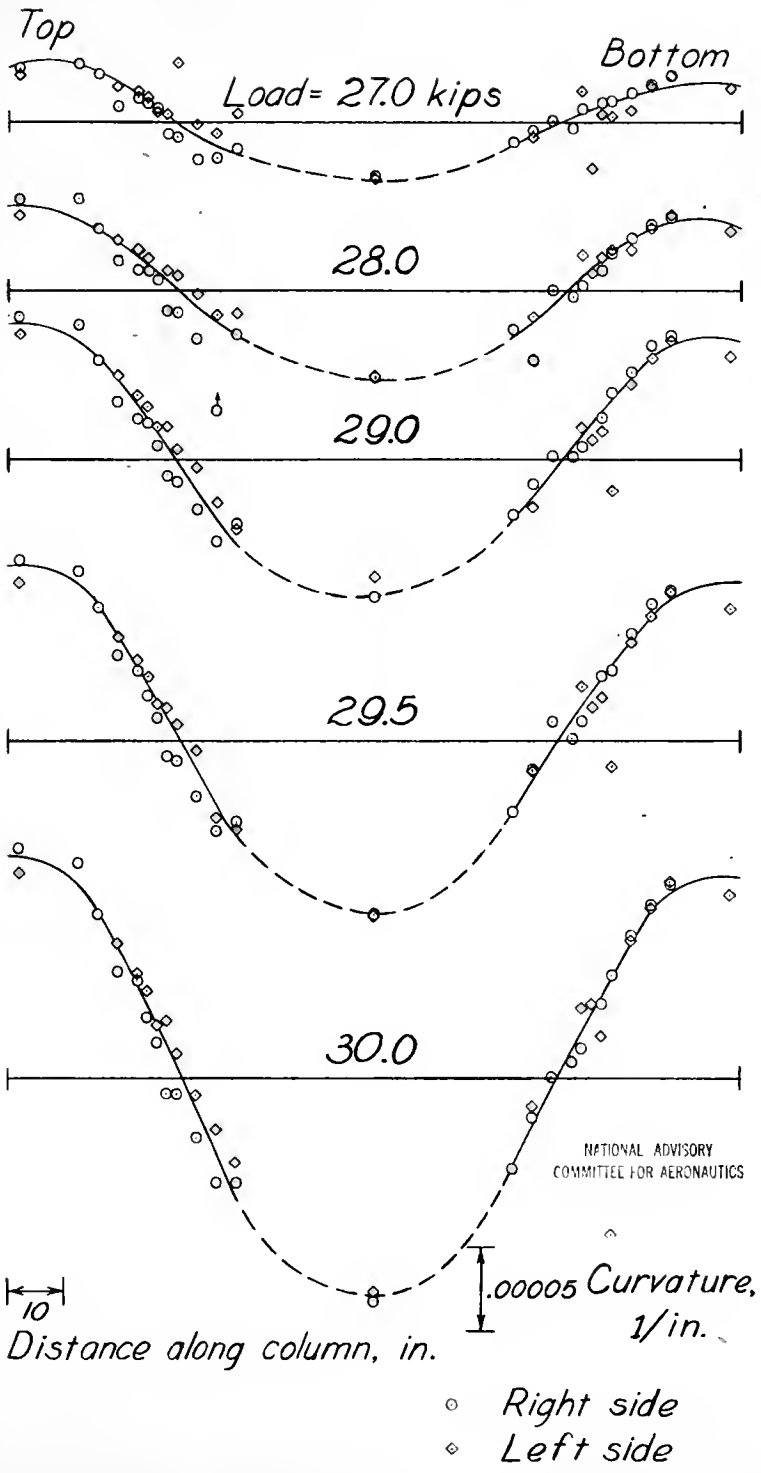
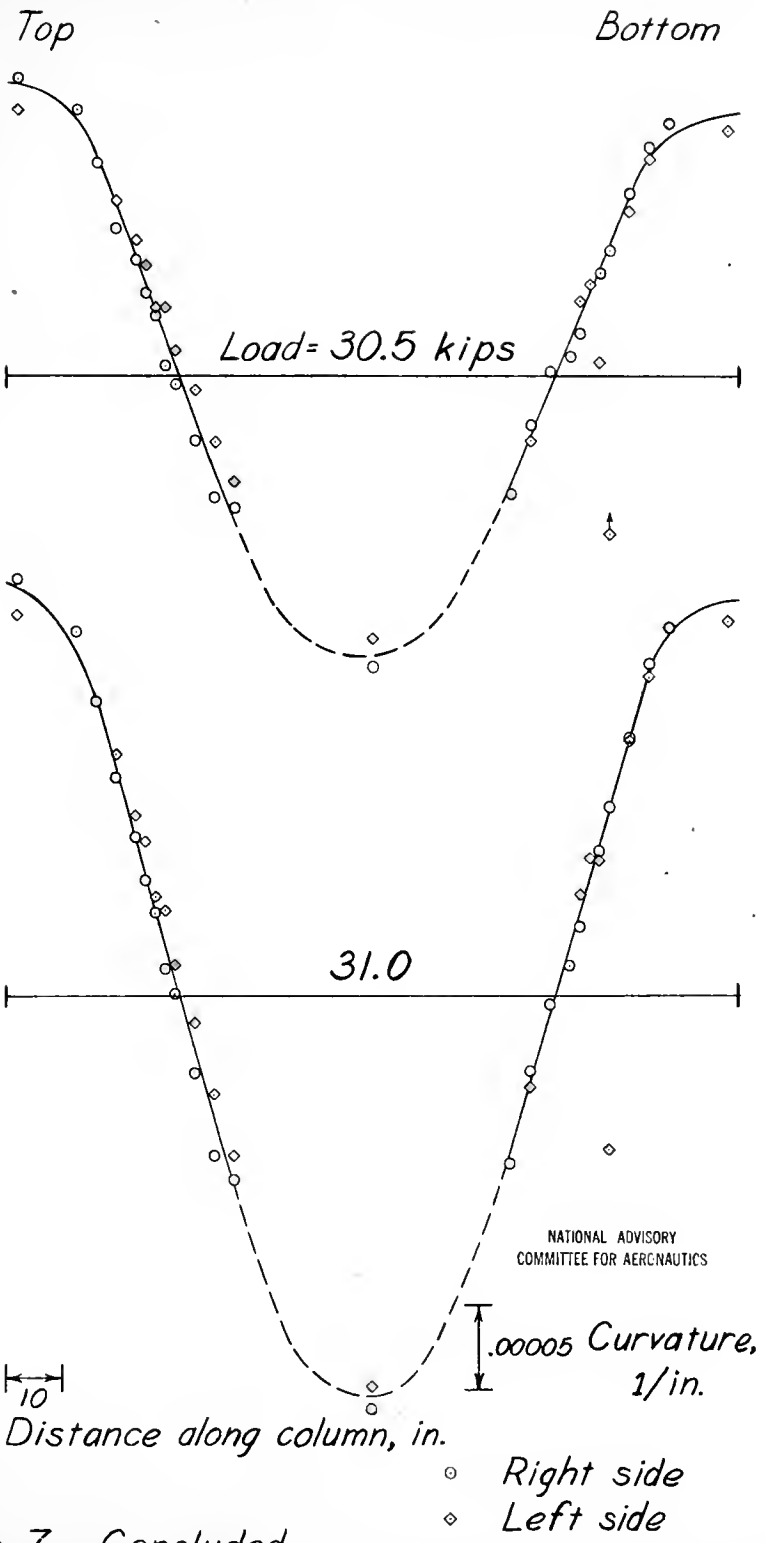


Figure 7.- Curvature of skin-and-stiffener column.



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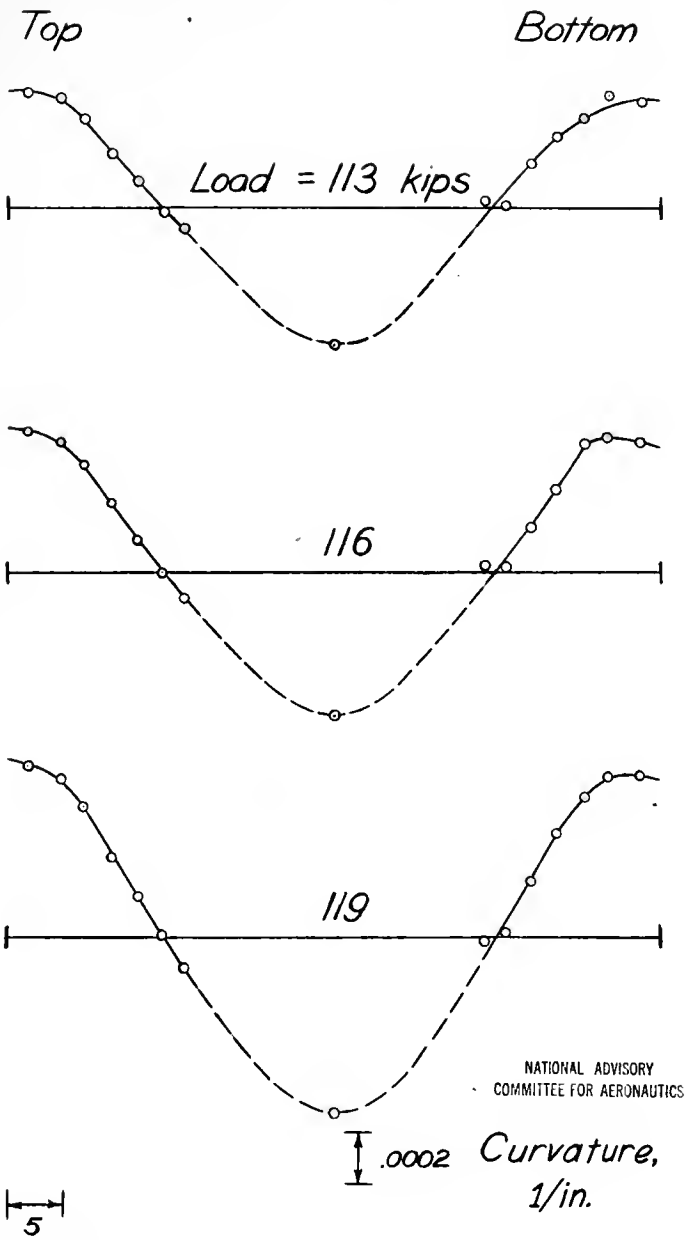

Figure 8.- Curvature of hat-stiffened panel column.

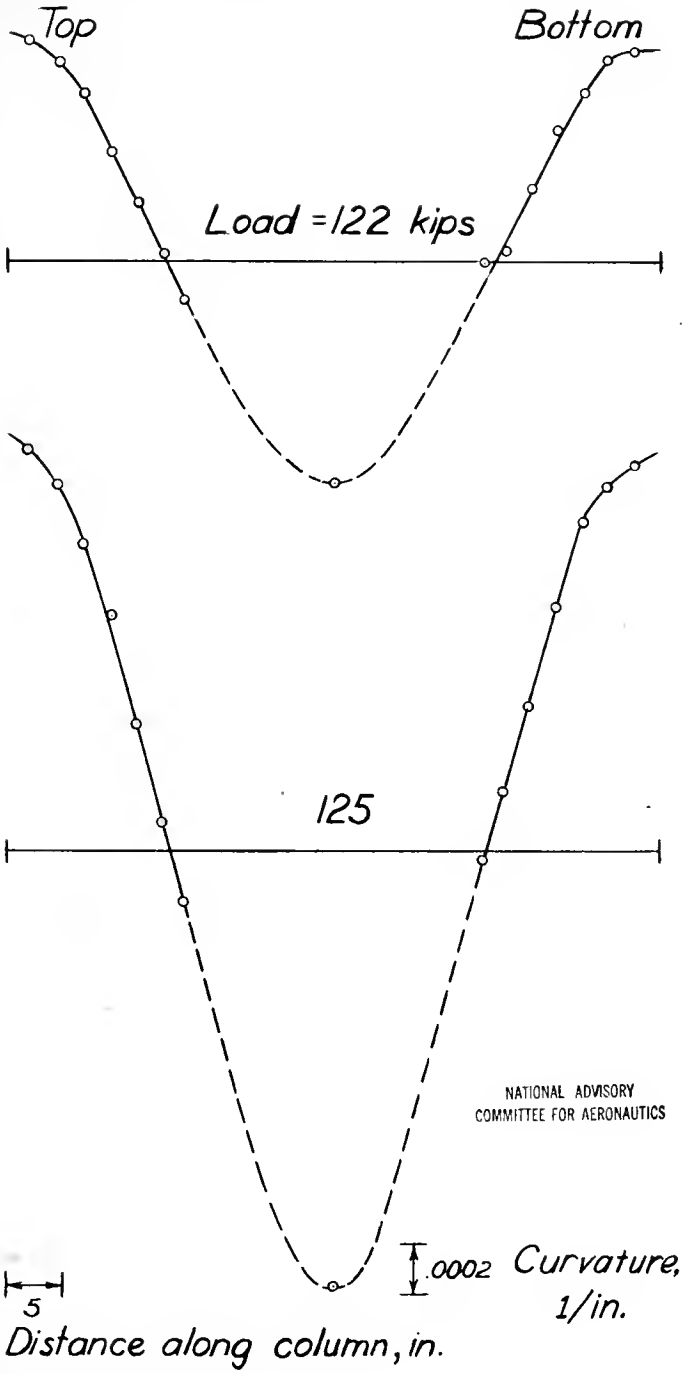

Figure 8.-Concluded.

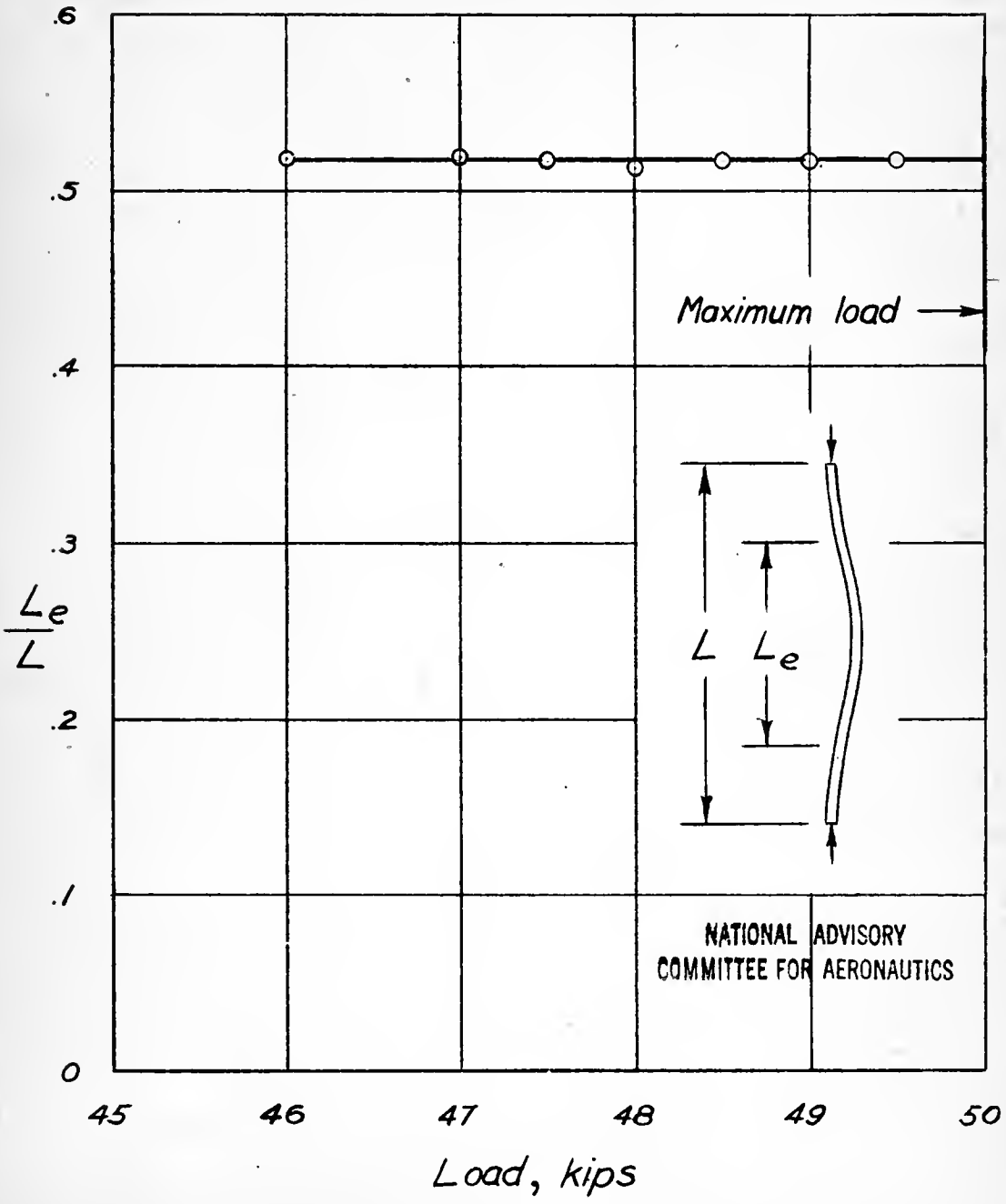

Figure 9.- Ratio of effective length to actual length for rectangular bar.

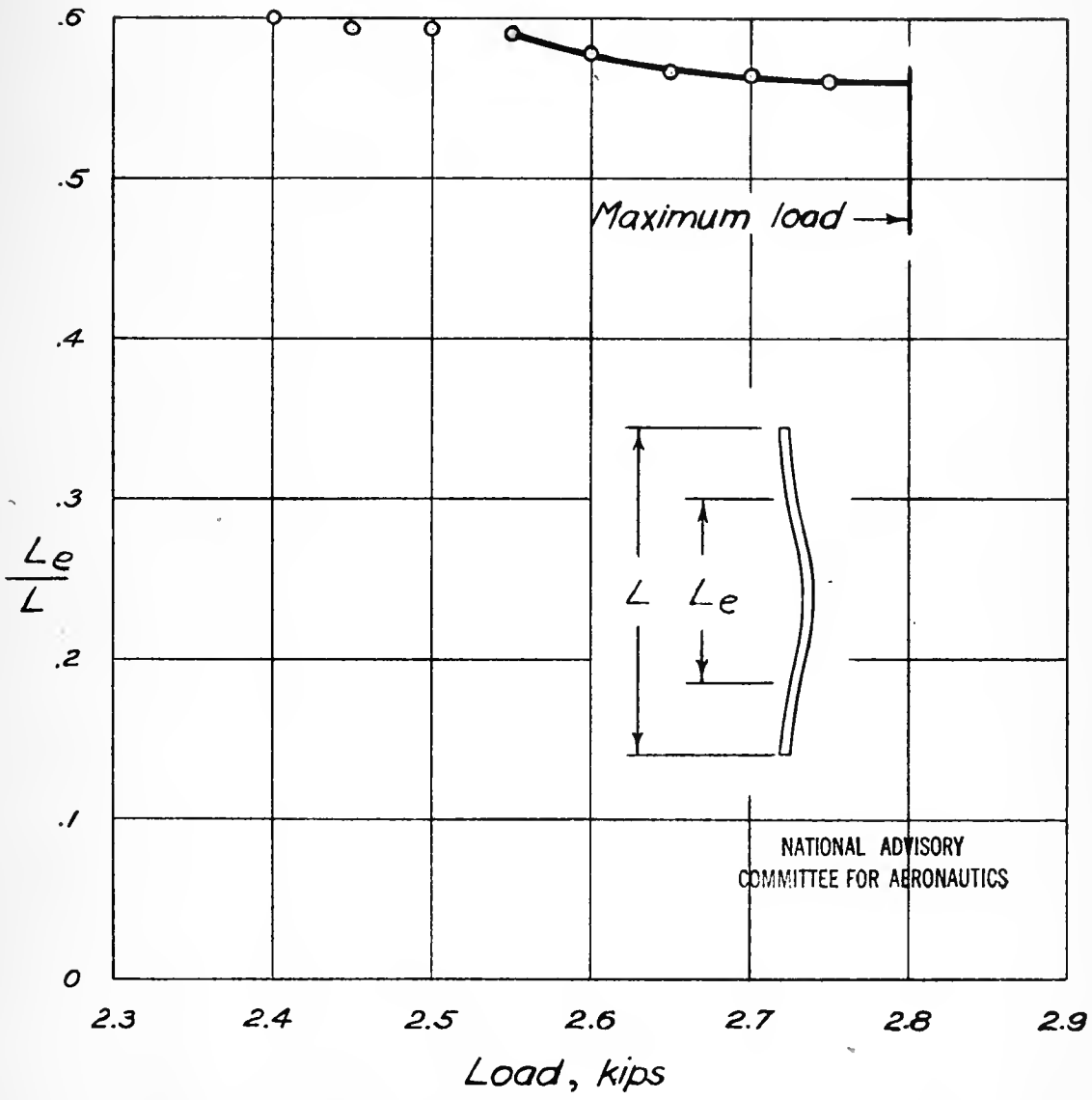

Figure 10.- Ratio of effective length to actual length for Z-section column.

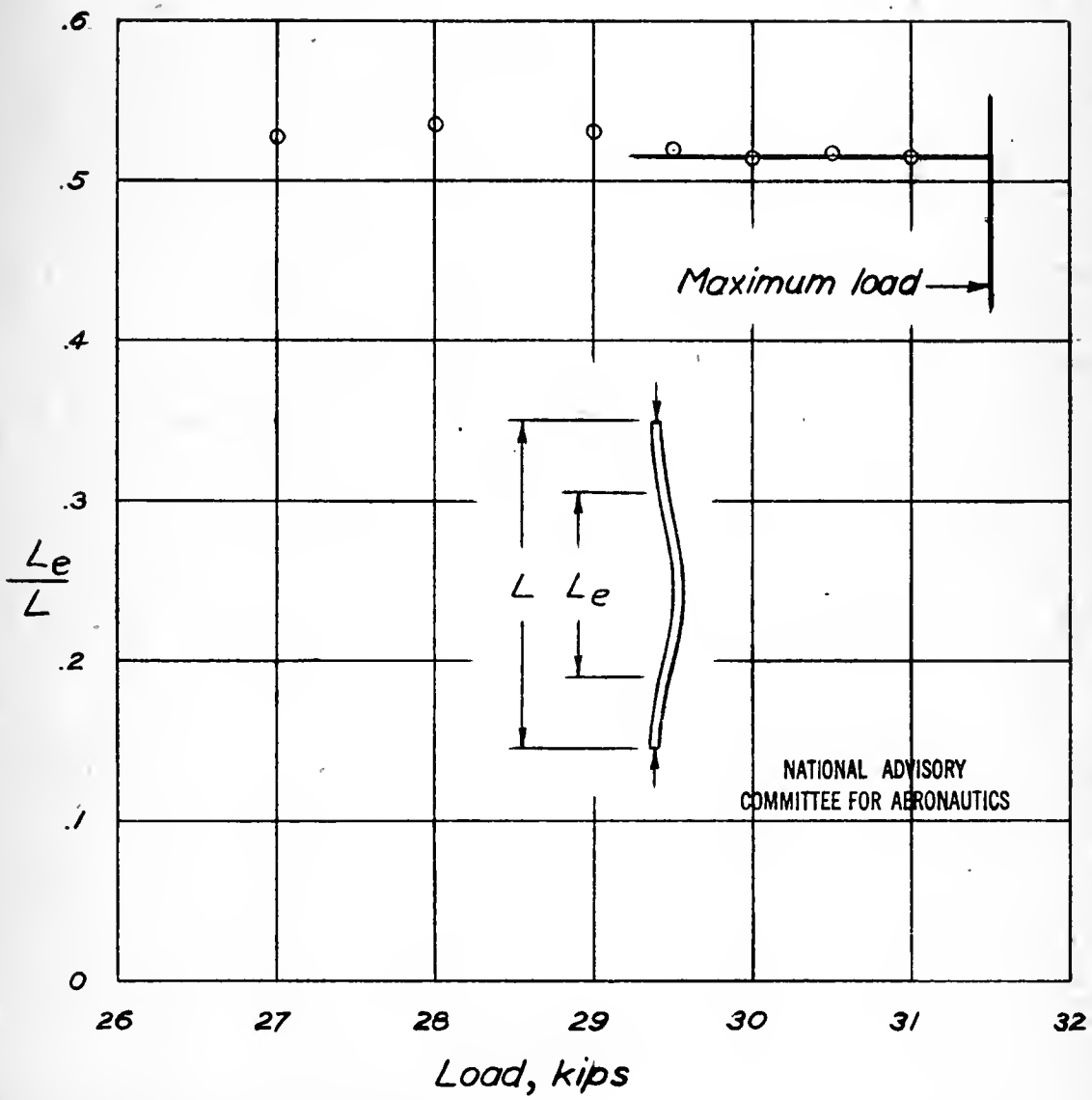

Figure 11.-Ratio of effective length to actual length for skin-and-stiffener column.

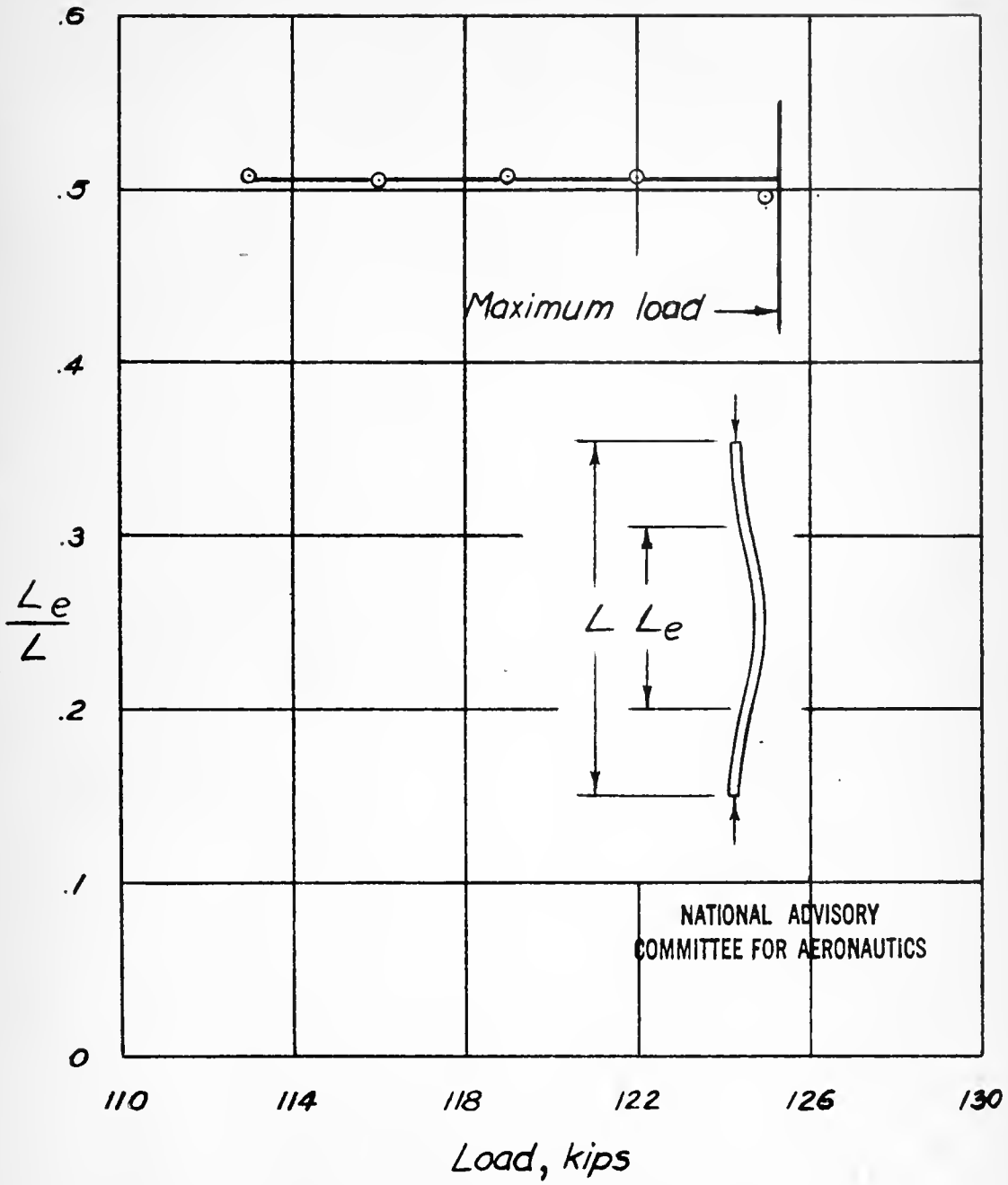

Figure 12.- Ratio of effective length to actual length for hat-stiffened panel column.

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